Secondary Special Camp 2011 : Geometry Exam

Total time alloted: 5 hours

[Each problem is worth 7 points. The problems are arranged in increasing difficulty.]

Problem 1. Let I be the incenter of triangle ABC. O_1 a circle passing through B and tangent to the line CI at I and O_2 a circle passing through C and tangent to the line BI at I. Prove that O_1, O_2 and the circumcircle of ABC pass through a single point.

Problem 2. The incircle of triangle ABC touches the sides BC, CA, AB at A', B', C' respectively. Let the midpoint of the arc AB of the circumcircle (not containing C) be C'', and define A'' and B'' similarly. Prove that the lines $A'A'', B'B'', C'C''$ are concurrent.

Problem 3. Let ABC be a triangle. Extend the side BC past C, and let D be the point on the extension such that $CD = AC$. Let P be the second intersection of the circumcircle of ACD with the circle with diameter BC. Let BP and AC meet at E, and let CP and AB meet at F. Prove that D, E, F are collinear.

Problem 4. Let ABC be an acute triangle and D, E, F the feet of its altitudes from A, B, C, respectively. The line through D parallel to EF meets line AC and line AB at Q and R, respectively. Let P be the intersection of line BC and line EF. Prove that the circumcircle of PQR passes through the midpoint of BC.

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