IMO MOCK - 1

Problem 1: Let n be a positive integer. Find all functions $f\colon\mathbb{R}\to\mathbb{R}$ such that, for all reals y and all nonzero reals x:

$$x^n f(y) - y^n f(x) = f\left(\frac{y}{x}\right)$$

Problem 2: Let ABCD be a convex quadrilateral with no pair of parallel sides, such that $\angle$ABC = $\angle$CDA. Assume that the intersections of the pairs of neighbouring angle bisectors of ABCD form a convex quadrilateral EFGH. Let K be the intersection of the diagonals of EFGH. Prove that the lines AB and CD intersect on the circumcircle of the triangle BKD.

Problem 3: The liar's guessing game is a game played between two players A and B. The rules of the game depend on two positive integers k and s which are known to both players.

At the start of the game A chooses integers x and N with 1 $\leq$ x $\leq$ N. Player A keeps x secret, and truthfully tells N to player B. Player B now tries to obtain information about x by asking player A questions as follows: each question consists of B specifying an arbitrary set S of positive integers (possibly one specified in some previous question), and asking A whether x belongs to S. Player B may ask as many questions as he wishes. After each question, player A must immediately answer it with yes or no, but is allowed to lie as many times as she wants; the only restriction is that, among any k+1 consecutive answers, at least one answer must be truthful.

After B has asked as many questions as he wants, he must specify a set X of at most n positive integers. If x belongs to X, then B wins; otherwise, he loses. Prove that:

1. If n $\geq 2^k$, then B can guarantee a win.

2. For all sufficiently large k, there exists an integer n$\geq (1.99)^k$ such that B cannot guarantee a win.

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