Mock-5

Day-1

i)Integers $a_0, a_1, a_2, \ldots . a_n$ are greater than or equal to -1 and are all non-zeros. If

$a_0 + 2a_1 + 2^2a_2 + \ldots . 2^n a_n = 0$, then prove that $a_0 + a_1 + a_2 + \ldots . + a_n > 0$

ii) Determine (with proof) all functions f : $[0,+\infty) \to [0,+\infty)$ such that for every x $\geq$ 0, we have

4f(x) $\geq$ 3x and f(4f(x)$-$3x) $=$ x.

iii) Let O and H be the circumcenter and orthocenter of acute $\triangle$ABC. The bisector of $\angle$BAC meets the circumcircle τ of $\triangle$ABC at D. Let E be the mirror image of D with respect to line BC. Let F be on τ such that DF is a diameter. Let lines AE and FH meet at G. Let M be the midpoint of side BC. Prove that GM$\perp$AF.

Day-2

iv)In how many ways can one choose n$-$3 diagonals of a regular n-gon, so that no two have an intersection strictly inside the n-gon, and no three form a triangle?

v)In acute $\triangle \mathrm{ABC}, \mathrm{AB} > \mathrm{AC}$. Let M be the midpoint of BC. The exterior angle bisector of $\angle$BAC meets ray BC at P. Points K and F lie on line PA such that MF$\perp$BC and MK$\perp$PA. Prove that BC^2=4PF.AK

vi) Let a, b, c and d be real numbers, and at least one of c or d is not zero. Let f:$\mathbb{R} \to \mathbb{R}$ be the function defined by f(x)=$\frac{\mathrm{ax}+b}{\mathrm{cx}+d}$. Assume that f(x) $\neq$ x for every x$\in\mathbb{R}$. Prove that if there exists at least one p such that f^{1387}(p) $=$ p, then for every x, for which f^{1387}(x) is defined, we have f^{1387} (x) $=$ x.