Set 1

- 1) Find all odd n such that F_n is divisible by 2011.
- 2) Points A_1, B_1, C_1 are selected inside triangle ABC on the altitudes from A, B, and C, respectively. [ABC] denotes the area of \triangle ABC. If

$$[ABC_1] + [BCA_1] + [CAB_1] = [ABC]$$

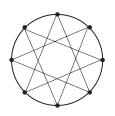
prove that the circumcircle of triangle $[A_1B_1C_1]$ passes through the orthocenter.

- 3) ¹ Consider a regular convex polygon marked with n- vertices dividing into n equal region. Now, choose a point and a number d and connect points d-th next point until you get to the point where you started. Now, the question is, how many regions are there now? The figure can be quite simple or complex. It depends on how you choose d. See the images below.
- 4) We are given a 2000-sided polygon in which no three diagonals are concurrent. Each diagonal is colored in one of 999 colors. Prove that there exists a triangle whose sides lie entirely on diagonals of one color. (The triangle's vertices need not be vertices of the 2000-sided polygon.)
- 5) A club has 42 members. Among each group of 31 members, there is at one pair of participants one male, one female who know each other. (Person A knows person B if and only if person B knows person A.) Prove that there exist 12 distinct males $a_1, a_2, ..., a_{12}$ and 12 distinct females $b_1, b_2, ..., b_{12}$ such that a_i knows b_i for all i.

^{1.} I haven't solved this myself yet. Just crossed my mind. So I am not sure how easy this is. :p









(a)

(b)

(c)

(d)