Some Angle-Related Problems

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Brilliant (Qi Huan Tan) P is a point in the interior of $\triangle ABC$ such that $\angle PBC = 30°, \angle PBA = 8°$, and, $\angle PAB = \angle PAC = 22°$. What is the measure of $\angle APC$?

Brilliant (Matt Enlow) The point M lies inside regular pentagon $ABCDE$ such that $\angle MBA = \angle MEA = 42°$. Find $\angle DMC$.

Brilliant (Sujoy Roy) ABC is an isosceles triangle with $AB = AC$ and $\angle BAC = 40°$. P is a point inside the triangle such that $\angle PBC = 30°$ and $\angle PCB = 50°$. Find the measure of $\angle APB$.

Brilliant (Azhagu Roopesh) In $\triangle ABC$, $AC = BC$ and $\angle ACB = 96°$. D is a point inside $\triangle ABC$ such that $\angle DAB = 18°$ and $\angle DBA = 30°$. What is the measure of $\angle ACD$?

Brilliant (Michael Ng) P is a point inside the circle (O), and, R is its inverse. AB is a chord of (O) passing through P such that $\angle AOB = 120°$. What is the measure of $\angle ARP$?

Brilliant (Dhruv Ramdev) In $\triangle ABC$, $\angle ABC = \angle ACB = 80°$. $D \in AB$ and $E \in AC$, such that, $\angle DCB = 60°$ and $\angle EBC = 70°$. What is the measure of $\angle DEB$?

Brilliant (Calvin Lin) In $\triangle ABC$, $\angle A = 30°$, $\angle B = 80°$. Point M lies inside the triangle, such that $\angle MAC = 10°$, $\angle MCA = 30°$. What is the measure of $\angle BMC$?

Brilliant (Calvin Lin) In $\triangle ABC$, $D \in AC$ such that $AD = DC$ and $\angle BDC \neq 90°$. Given that $\angle DCB = 25°$ and $\angle DBA = 65°$, what is the measure of $\angle BDC$?

Brilliant (Xuming Liang) In $\triangle ABC$, $\angle A = 80°$, $\angle B = 65°$, $\angle C = 35°$. The internal angle bisectors of $\angle B, \angle C$ meet their respective opposite sides at E, F. Let B' be a point on segment AC such that $AB' = AB$. Let the circumcircle of $BB'E$ intersects FE at J. Find the measure of $\angle AJB$.

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Brilliant (Calvin Lin) In a triangle ABC, $\angle A = 84°$, $\angle C = 78°$. Points D and E are taken on the sides AB and BC, so that $\angle ACD = 48°$, $\angle CAE = 63°$. What is the measure of $\angle CDE$?

Brilliant (Calvin Lin) $\triangle ABC$ has angles $\angle A = 3\alpha$, $\angle B = 90°$ and $\angle C = 3\gamma$. Let, P be a point on segment BC such that $\angle PAB = \alpha$, and, Q be a point on segment BA such that $\angle QCB = \gamma$. Let R be a point within the triangle such that $\angle RAC = \alpha$ and $\angle RCA = \gamma$. If $\angle QRC = 142°$, what is the measure of α?

JBMO 2007/2 $ABCD$ is a convex quadrilateral with $\angle DAC = \angle BDC = 36°, \angle CBD = 18°$ and $\angle BAC = 72°$. The diagonals of $ABCD$ intersect at P. What is the measure of $\angle APD$?

USAMO 1996/5 $\triangle ABC$ has the following property: there is an interior point P such that $\angle PAB = 10°, \angle PBA = 20°, \angle PCA = 30°$, and $\angle PAC = 40°$. Prove that $\triangle ABC$ is isosceles.

G. Revisited Let ABC be an isosceles triangle with $\angle B = \angle C = 80°$. Cevians BD and CE of $\triangle ABC$ are drawn such that $\angle DBC = 60°$ and $\angle ECB = 50°$. Find the measure of $\angle EDB$.

G. Revisited ABCD is a square, and, P a point inside it such that $\angle PAB = \angle PBA = 15°$. Prove that, the points P, C, D are the vertices of an equilateral triangle.

ILL'71 Diagonals of a convex quadrilateral $ABCD$ intersect at O. If $\angle OBA = 30°$, $\angle OCB = 45°, \angle ODC = 45°$, and, $\angle OAD = 30°$, find all the angles of the quadrilateral.

IMO 1975/3 On the sides of an arbitrary $\triangle ABC$, $\triangle^s BPC, CQA, ARB$ are erected externally such that,

$$\angle PBC = \angle CAQ = 45°,$$
$$\angle BCP = \angle QCA = 30°,$$
$$\angle ABR = \angle BAR = 15°.$$

Prove that, $\angle QRP = 90°$, and, $QR = RP$.

ILL'74 Outside an arbitrary triangle ABC, triangles ADB and BCE are constructed such that $\angle ADB = \angle BEC = 90°$ and $\angle DAB = \angle EBC = 30°$. On the segment AC, the point F is chosen such that, $AF = 3FC$. Prove that,

$$\angle DFE = 90° \quad \text{and} \quad \angle FDE = 30°.$$

ILL'77 In $\triangle ABC, \angle A = 30°$ and $\angle C = 54°$. $D \in BC$ and $E \in AB$ such that $\angle CAD = 12°$ and $\angle ACE = 6°$. $AD \cap CE = S$. Prove that, $BS = BC$.

ISL'79 Inside an equilateral triangle ABC, points P, Q, R are constructed such that,

$$\angle QAB = \angle PBA = 25°$$

$$\angle RBC = \angle QCB = 20°$$

$$\angle PCA = \angle RAC = 25°$$

Determine the angles of $\triangle PQR$.

ILL'84 Let ABC be an isosceles triangle, where, $AB = AC$ and $\angle A = 20°$. Let, D be a point on AB, and E a point on AC, such that, $\angle ACD = 20°$ and $\angle ABE = 30°$. What is the measure of $\angle CDE$?

ILL'86 In $\triangle ABC, \angle BAC = 100°$, and, $AB = AC$. $D \in AC$ such that, $\angle ABD = \angle CBD$. Prove that, $AD + DB = BC$.[1]

ILL'88 Let, E be a point external to a circle Γ. Two chords EAB and ECD meet at an angle of $40°$ such that the points A, B, C, D lie on Γ. If $AB = BC = CD$, find the measure of $\angle ACD$.

ILL'88 In $\triangle ABC, AB = BC$, and, $\angle CAB = 80°$. Points D, E, F lie on the sides BC, CA, AB respectively, such that, $CE = CD$, and, $BF = FD$. find the measure of $\angle EDF$.

ISL'92 In $\triangle ABC$, D, E are the intersections of the bisectors of $\angle ABC, \angle ACB$ with AC, AB respectively. If $\angle BDE = 24°$ and $\angle CFD = 18°$, determine the angles of $\triangle ABC$.

Sources:

[1] The IMO Compendium

[2] Geometry Revisited, by, H.S.M. Coxeter

[3] Brilliant: https://brilliant.org/

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[1]Co-incidentally, this was set as a problem in last year's Math Olympiad of Holy-Cross Science Festival